

Seat No./Enrollment No.: _____

GUJARAT TECHNOLOGICAL UNIVERSITY
BCA/MCA INTEGRATED – SEMESTER 3 – EXAMINATION – SUMMER 2026

Subject Code: BC03001041

Date: 21/05/2026

Subject Name: MATHEMATICAL FOUNDATION FOR AI

Time: 02:30 PM TO 05:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of simple calculators and non-programmable scientific calculators are permitted.

- Q.1** (a) 1. Define identity matrix. 07
2. Define independent events.
3. State the meaning of probability.
4. Describe n-ary Relations.
5. What is optimization problem?
6. If X and Y are independent random variables on a sample space S, then $E(XY) = E(X)E(Y)$. (True/False)
7. State the law of large numbers.
(b) Explain Singular Value Decomposition with suitable example. 07
- Q.2** (a) Find out inverse and transpose of following matrix: 07
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(b) Differentiate discrete and continuous probability distribution functions. 07
OR
(b) Find out eigenvalues and eigenvectors of following matrix: 07
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$
- Q.3** (a) Discuss Bayes Theorem in brief. 07
(b) Discuss the concept of gradient descent in brief. 07
OR
- Q.3** (a) Discuss the characteristics of Lagrange multiplier. 07
(b) Let $R_1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Find following: 07
1) $R_1 \cup R_2$
2) $R_1 \cap R_2$
3) $R_1 - R_2$
4) $R_2 - R_1$
- Q.4** (a) Construct the truth table of the proposition $(p \vee \neg q) \rightarrow (p \wedge q)$. 07
(b) Discuss the rules of inference for propositional logic. 07
OR
- Q.4**

- (a) List the members of following sets. 07
- a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
 - b) $\{x \mid x \text{ is a positive integer less than } 12\}$
 - c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$
 - d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$
- (b) Define and discuss convex function in detail. 07
- Q.5** (a) Discuss the symmetric and reflective properties of relation with suitable example. 07
- (b) Construct a truth table for $((p \rightarrow q) \rightarrow r) \rightarrow s$. 07
- OR
- Q.5** (a) Discuss the applications of mathematics in AI. 07
- (b) "For probabilistic models, it is mathematically convenient to use $\{0, 1\}$ as a binary representation". Discuss the statement with suitable example. 07

Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY
BCA/MCA INTEGRATED - SEMESTER - III EXAMINATION - WINTER 2025

Subject Code: BC03001041

Date: 05-12-2025

Subject Name: Mathematical Foundation for AI

Time: 10:30 AM TO 01:00 PM

Total Marks: 70

Instructions

- 1. Attempt all questions.**
- 2. Make suitable assumptions wherever necessary.**
- 3. Figures to the right indicate full marks.**
- 4. Use of simple calculators and non-programmable scientific calculators are permitted.**

Marks

Q.1 (a) Do as directed :

07

- 1) What is Vector ?
- 2) Full form of SVD
- 3) What is Unit matrix ?
- 4) Find the power set of $A = \{1, 2, 9\}$
- 5) What is Universal set ?
- 6) What does the Law of Large Numbers state?
- 7) What is Diagonal matrix ?

(b)

Find the eigenvalues and the eigenvector for the matrix $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$

07

Q.2 (a) Let A and B are two events of an experiment such that $p(A) = 3/10$, $p(B) = 1/2$, $p(B/A) = 2/5$, then find :

07

- 1) $p(A \cap B)$
- 2) $p(A/B)$
- 3) $p(A \cup B)$

(b) Find X and Y if $2x - 3y = 15$ and $x + 5y = -18$ using Cramer's rule.

07

OR

(b)

If $A = \begin{bmatrix} 3 & 2 & -1 \\ 5 & 4 & -3 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 9 & 3 \\ 0 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix}$ Compute : $2A + 5B$, $(B+A)^T$, $A - B$.

07

Q.3 (a) A bag I contains 4 white and 6 black balls while another bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags and it is found to be black. Find the probability using Baye's theorem that it was drawn from bag I.

07

(b) The dataset below represents the scores of 5 students in a quiz: $\{5, 7, 9, 11, 13\}$. Calculate mean and Variance.

07

OR

- (a) Suppose 2 dice are tossed and the number on the top face recorded what is the probability that : 07

- 1) Even number was tossed
- 2) Sum of number is less than 5
- 3) Prime number was tossed
- 4) Sum of number is atleast 7
- 5) At least one of the dice shows up a 3
- 6) Getting a doublet of even numbers
- 7) Getting six as a product

- (b) Find the variance and standard deviation of the following data: 07

X	5	10	15	20	25	30
f(x)	6	7	3	2	1	1

- Q.4** (a) $P = \{x \in \mathbb{N} / 2 < x \leq 8\}$, $Q = \{x \in \mathbb{N} / 0 \leq x < 5\}$, $R = \{x \in \mathbb{N} / 1 \leq x \leq 10\}$, $U = \{x \in \mathbb{Z} / -2 \leq x < 12\}$ 07
Verify $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$

- (b) Prepare the truth table of the propositional statement $(p \wedge (\sim q \vee \sim r)) \Rightarrow (p \Rightarrow \sim q)$ 07

OR

- (a) $A = \{2, 5, 7, 18\}$, $B = \{2, 3, 4, 5, 10\}$, $R = \{a \leq b\}$. Find graph, matrix and inverse of relation. 07

- (b) If $2x^2 - 4x - 6 = 0$, Evaluate using Quadratic function formula. 07

- Q.5** (a) The probability of a student passing in Science is $\frac{4}{5}$ and the student passing in both science and maths is $\frac{1}{2}$. What is probability of that student passing in maths knowing that he passed in science ? 07

- (b) Find the value of x if $\log_3 x = \log_3^4 + \log_3^7$. 07

OR

- (a) A fair six-sided die is rolled repeatedly. According to the law of large number what is the expected average value of the outcomes as the number of rolls increases ? 07

- (b) Find the derivative of $x^6 + x^3 + 2$. 07
